

# Modelling and Performance analysis of a Network of Chemical Sensors with Dynamic Collaboration

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## Abstract

The problem of environmental monitoring using a wireless network of chemical sensors with a limited energy supply is considered. Since the conventional chemical sensors in active mode consume vast amounts of energy, an optimisation problem arises in the context of a balance between the energy consumption and the detection capabilities of such a network. A protocol based on “dynamic sensor collaboration” is employed: in the absence of any pollutant, majority of sensors are in the sleep (passive) mode; a sensor is invoked (activated) by wake-up messages from its neighbors only when more information is required. The paper proposes a mathematical model of a network of chemical sensors using this protocol. The model provides valuable insights into the network behavior and near optimal capacity design (energy consumption against detection). An analytical model of the environment, using turbulent mixing to capture chaotic fluctuations, intermittency and non-homogeneity of the pollutant distribution, is employed in the study. A binary model of a chemical sensor is assumed (a device with threshold detection). The outcome of the study is a set of simple analytical tools for sensor network design, optimisation, and performance analysis.

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## 1. Introduction

Development of wireless sensor network (WSN) for a particular operation scenario is a complex scientific and technical problem [1], [2]. Very often this complexity resides

in establishing a balance between the peak performances of the WSN prescribed by the operational requirements (e.g. minimal detection threshold, size of surveillance region, detection time, rate of false negatives, etc) and various resource constraints (e.g. limited energy supply, limited number of sensors, limited communication range, fixed detection threshold of individual sensors, limited budget for the cost of hardware, maintenance, etc). The issue of resource constraints becomes even more relevant for a network of chemical sensors that are used for the continuous environmental monitoring (air and water pollution, hazardous releases, smoke etc). The reason is that a modern chemical sensor is usually equipped with a sampling unit (a fan for air and a pump for water), which turns on when the sensor is active. The sampling unit usually requires a significant amount of energy to operate as well as frequent replacement of some consumable items (i.e. cartridges, filters). This leads to the critical requirement in the design of a WSN to reduce the active (i.e. sampling) time of its individual sensors.

One attractive way to achieve an optimal balance between the peak performance of the WSN and its constraints in resources, mentioned above is to exploit the idea of Dynamic Sensor Collaboration (DSC) [3], [4]. The DSC implies that a sensor in the network should be invoked (or activated) only when the network will gain information by its activation [4]. For each individual sensor this information gain can be evaluated against other performance criteria of the sensor system, such as the detection delay or detection threshold, to find an optimal solution in the given circumstances.

While the DSC-based approach is a convenient framework for the development of algorithms for optimal scheduling of constrained sensing resources, the DSC-based algorithms involve continuous estimation of the state of each sensor in the network and usually require extensive computer simulations [3], [4]. These simulations may become unpractical as the number of sensors in the network increases (e.g. “smart dust” sensors). Even when feasible, the simulations can provide only the numerical values for optimal network parameters, which are specific for an analysed scenario, but without any analytical framework for their consistent interpretation and generalisation. For instance, the scaling properties of a network (the functional relationship between the network parameters) still remain undetermined, which prevents any comprehensive optimisation study.

This motivates the development of another, perhaps less rigorous, but certainly simpler approach to the problem of network analysis and design. The main idea is to phenomenologically employ the so-called bio-inspired (epidemiology, population dynamics) or physics inspired (percolation and graph theory) models of DSC in the sensor network in order to describe the dynamics of collaboration as a single entity [5], [6], [7], [9], [10],[11]. Since the theoretical framework for the bio- or physics- inspired models is already well established, we are in the position to make significant progress in the analytical treatment of these models of DSC (including their optimisation). From a formal point of view the derived equations are ones of the “mean-field” theory, meaning that instead of working with dynamic equations for each individual sensor we only have a small number of equations for the “averaged” sensor state (i.e. passive, active, faulty etc), *regardless of the number of the sensors in the system*. A revealing example of the efficiency of this approach is the celebrated SIR model in epidemiology [12]. For any size of population, the SIR model describes the spread of an infection by using only three equations, corresponding to three “infectious” classes of the population: susceptible, infectious and recovered.

The analytic or “equation-based” approach, often leads to valuable insights into the performance of the proposed sensor network system by providing simple analytical expressions to calculate the vital network parameters, such as detection threshold, robustness, responsiveness and stability and their functional relationships.

In the current paper we develop a simple model of a wireless network of chemical sensors, where dynamic sensor collaboration is driven by the level of concentration of a pollutant (referred to as the “external challenge”) at each individual sensor. Our approach is based on the known analogy [11] between the information spread in a sensor network and the epidemics propagation across a population. In this analogy, the infection transmission process corresponds to message passing among the sensors. A chain reaction in transmission of an infection is called the epidemic. In the context of a sensor network, a chain reaction will trigger the network (as a whole) to move from the “no pollutant” state to the “pollutant present” state, which will indicate the presence of an external challenge.

The paper shows that the adopted epidemics or population inspired approach can

provide a reliable description of the dynamics of such a sensor network. The simple analytical formulas (scaling laws) derived from the model express the relationships between the parameters of the network (e.g. number of sensors, their density, sensing time etc), the network performance (probability of detection, response time of a network) and the parameters of the external challenge (environment, pollutant). As an example of application of the proposed framework we performed a simple optimisation study. Numerical simulations are carried out and presented in the paper in support of analytical expressions.

Although the model presented in this paper is specific to a network of chemical sensors, the underlying analytical approach can be easily adapted to other applications and other types of networks by a simple change of the model of environment and sensor.

## 2. The Model of Environment

The external challenges are modeled by a random time series which mimics the turbulent fluctuation of concentration at each sensor of the network. In this approach the fluctuations in concentration  $C$  are modeled by the probability density function (pdf) of  $C$  with the mean  $C_0$  as a parameter (i.e.  $C_0$  is a mean concentration of the tracer in the area) [13] :

$$f(C|C_0) = (1 - \omega)\delta(C) + \frac{\omega^2 (\gamma - 1)}{C_0 (\gamma - 2)} \left(1 + \frac{\omega}{(\gamma - 2)} \frac{C}{C_0}\right)^{-\gamma}. \quad (1)$$

Here the value  $\gamma = 26/3$  can be chosen to make it compliant with the theory of tracer dispersion in Kolmogorov turbulence (see [13]), but it may vary with the meteorological conditions. The parameter  $\omega$ , which models the tracer intermittency in the turbulent flow, can be in the range  $[0, 1]$ , with  $\omega = 1$  corresponding to the non-intermittent case. In general it also depends on a sensor position within a chemical plume, thus  $\omega$  is in the range  $0.95 - 0.98$  near the plume centroid and may drop to  $0.3 - 0.5$  near the plume edge. For  $\omega \neq 0$ , the pdf  $f$  of (1) has a delta impulse in zero, meaning that the measured concentration in the presence of intermittency can be zero on some occasions. It can be easily shown that the pdf of (1) integrates to unity, so it is appropriately normalized.

The measured concentration time series can be generated by drawing random samples from the probability density function given in (1) at each time step. The random number

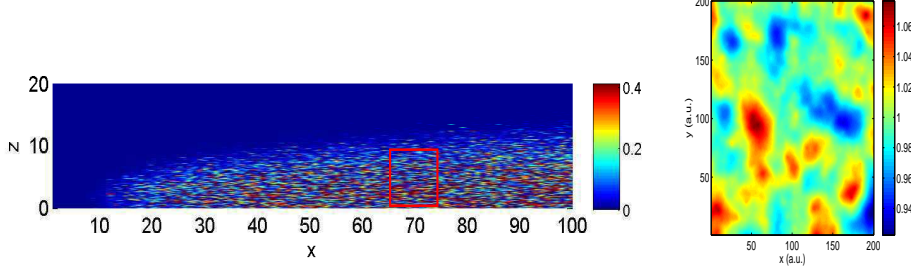


Figure 1: Example for concentration realisation for a plume-like flow (left) and the selected area of the flow with higher resolution (right).

generator is implemented using the *inverse transform* method based on the following steps [8] :

1. Draw a sample  $u$  from the standard uniform distribution:  $u \sim U[0, 1]$ .
2. Compute the value of  $C$  that satisfies  $F(C) = u$ , where  $F(\cdot)$  is the cumulative distribution function (cdf) of the distribution of interest.
3. The value of  $C$  computed in the previous step is a random sample drawn from the desired probability distribution.

The cdf  $F(\cdot)$  needed for inverse transform sampling is obtained by integrating the pdf in (1), and is given by:

$$F(C|C_0) = 1 - \omega \left[ 1 + \left( \frac{2}{\gamma - 2} \right) \frac{C}{C_0} \right]^{1-\gamma}. \quad (2)$$

The use of this cdf in the inverse transform sampling procedure generates the value of concentration:

$$C = \begin{cases} C_0 \left( \frac{\gamma-2}{\omega} \right) \left[ \left( \frac{1-u}{\omega} \right)^{-\frac{1}{\gamma-1}} - 1 \right], & u \geq 1 - \omega \\ 0, & u < 1 - \omega, \end{cases} \quad (3)$$

where  $u$  is again the standard uniform distribution  $u \sim U[0, 1]$ .

In order to produce spatial correlations that comply with the well-known scaling properties of turbulent dispersion a special ‘swapping’ algorithms was implement. This recursive algorithm mimic the chaotic fluctuations occurring in the real turbulent flows (for details, see [16]).

The proposed framework allows to implement a reasonably realistic model of the contaminated environment (i.e to generate the concentration realisation at each sensor over

time), see Fig 1. Due to a universal nature of turbulence it can be used to simulate performance of WSN in detection of either airborne and waterborne releases. The parameters  $\gamma$  and  $\omega$  are typically estimated from geophysical observation (meteorological and organological) and will be assumed known.

The geometrical complexity of the turbulent flow can be incorporated in the theoretical framework (2) by assuming a temporal and spatial variability of the mean concentration field  $C_0 \equiv C_0(\mathbf{r}, t)$ . This way we can simulate various morphologies of the flow (jet, wake, boundary layer, compartment flow, etc) as well as various scenarios of hazardous release (plume, puff), for details see [8], [17]. For the sake of simplicity in the current paper we consider only case  $C_0 = \text{const}$ . This assumption corresponds to the approximation when the size of WSN is less than the width of hazardous plume (see Fig 1), or to an important practical case of a ‘highly distributed’ source of pollutant (traffic, extended industrial site or urban area [18]).

### 3. The Model of a Chemical Sensor

We adopt a simple binary (or “threshold”) model of a sensor, with the sensor reading  $V$  given by:

$$V = \begin{cases} 1, & C \geq C_* \\ 0, & C < C_*. \end{cases} \quad (4)$$

We emphasize that threshold  $C_*$  is an internal characteristic of the sensor, unrelated to  $C_0$  in (1). This threshold is another important parameter of our model. A chemical sensor with bar readings, which includes many subsequent levels for concentration thresholds mapped into a discrete sensor output, is an evident generalisation of (4).

Using (3) and (4) it is straightforward to derive the probability of detection for an individual sensor embedded in the environment characterised by (2):

$$p = 1 - F(C_*|C_0). \quad (5)$$

This aggregated parameter links the characteristics of a specific sensor  $C_*$ , the parameter of the external challenge  $C_0$  and the environment  $(F(\cdot), \gamma, \omega)$ .

#### 4. Modeling and Analysis of Network Performance

Our focus is a wireless network of chemical sensors with dynamic collaboration. We assume that  $N$  identical sensors (i.e with the same detection threshold  $C_*$  and sampling time  $\tau_*$ ) are uniformly distributed over the surveillance domain of area  $S$  with density  $\rho = N/S$ .

We will model the following network protocol for dynamic collaboration. Each sensor can be only in one of the two states: *active* or *passive*. The sensor can be activated only by a message it receives from another sensor. Once activated, the sensor remains in the active state during an interval of time  $\tau_*$ ; then it returns to the passive (sleep) state. While being in the active state, the sensor senses the environment and if the chemical tracer is detected (binary detection), it broadcasts a (single) message. If a sensor receives an activation message while it is in the active state, it will ignore this message. The broadcast capability of the sensor is characterized by its communication range  $r_*$ , which is another important parameter of the model. The described protocol assumes that certain sensors of the network are permanently active. The number of permanently active sensors in the network is fixed but the actual permanently active sensors vary over time in order to equally distribute the energy consumption of individual sensors.

The WSN following this protocol can be considered as a system of agents, interacting with each other (by means of message exchange) and with the stochastic environment (by means of sampling and probing). The interactions can change the state of agents (active and passive). From this perspective this WSN is similar to the epidemic SIS (susceptible-infected-susceptible) model [12], in which an individual can be in only two states (susceptible or infected), and the change of state is a result of interaction (mixing) between the individuals (which corresponds to the exchange of messages in our case). Thus a dynamic (population) model for our system [12] is as follows:

$$\frac{dN_+}{dt} = \alpha N_+ N_- - \frac{N_+}{\tau_*}, \quad (6)$$

$$\frac{dN_-}{dt} = -\alpha N_+ N_- + \frac{N_+}{\tau_*}, \quad (7)$$

where  $N_+$ ,  $N_-$  denote the number of active and passive sensors, respectively. The nonlinear terms on the RHS of (6) and (7) are responsible for the interaction between individuals

(i.e. sensors), with the parameter  $\alpha$  being a measure of this interaction. The population size (i.e. the number of sensors) is conserved, that is  $N_+ + N_- = N = \text{const.}$

The next step is to express  $\alpha$  in terms of the parameters of our system by invoking physics based arguments used in population dynamics [12]. It is well-known that parameter  $\alpha$  in (6) describes the intensity (contact rate) of social interaction between individuals in the community, so we can propose (see [12], [15])

$$\alpha \propto \frac{mp}{N\tau_*}, \quad (8)$$

where  $m$  is the number of contacts made by an “infected” sensor during the infectious period  $\tau_*$  (i.e the number of sensors receiving a message from an alerting sensor). In our case we have  $m = \pi r_*^2 \rho$ . Then using  $N = S\rho$  we can write

$$\alpha = G \frac{\pi r_*^2}{\tau_* S} p, \quad (9)$$

where  $G$  is a constant calibration factor, being of order unity (it must be estimated during the network calibration);  $p$  was defined by (5). In order to simplify notation, from now on we will assume that  $G$  is absorbed in the definition of  $r_*$ .

It is worth noting that by introducing non-dimensional variables  $n_+ = N_+/N, n_- = N_-/N, \tau = t/\tau_*$  the system (6)-(7) can be rewritten in a compact non-dimensional form

$$\frac{dn_+}{d\tau} = R_0 n_+ n_- - n_+, \quad n_- = 1 - n_+, \quad (10)$$

with only one non-dimensional parameter

$$R_0 = \alpha \tau_* N. \quad (11)$$

The parameter  $R_0$  is well-known in epidemiology where it has the meaning of a *basic reproductive number* [12].

The system (6)-(7) combined with the condition  $N_+ + N_- = N$  can be reduced to one equation for  $y = N_+$

$$\frac{dy}{dt} = \alpha y(N - y) - \frac{y}{\tau_*} = y(b - \alpha y), \quad (12)$$

where

$$b = \alpha N - 1/\tau_* = (R_0 - 1)/\tau_*. \quad (13)$$



By simple change of variables  $z = \alpha y/b$  this equation can be reduced to the standard logistic equation

$$\frac{dz}{dt} = bz(1 - z), \quad (14)$$

which has the well-known solution

$$z(t) = \frac{z_0}{(1 - z_0) \exp(-bt) + z_0}, \quad (15)$$

where  $z_0 = z(0)$ .

We can see that if  $b < 0$  then  $z \rightarrow 0$  as  $t \rightarrow \infty$  for any  $z_0$ , so any individual sensor activation in the network will “die out”, that is the network will not be able to detect the external challenge. The same is valid for  $b = 0$  when  $z = z_0 = \text{const}$  (no response to external challenges). Only if the condition  $b > 0$  is satisfied, then  $z \rightarrow 1$  as  $t \rightarrow \infty$  (independently of  $z_0$ ). In this case, after a certain transition interval, the network will reach a new steady state with

$$\frac{N_+}{N} = 1 - \theta, \quad \frac{N_-}{N} = \theta, \quad \theta = \frac{1}{\alpha\tau_*N} \equiv \frac{1}{R_0}. \quad (16)$$

A fraction of active sensors  $N_+$  at this new state is a measure of the network (positive) response to the event of chemical contamination. From (15) it is clear that the time scale for the network to reach the new state can be estimated from the condition  $e^{-bt} \ll 1$ , so

$$\tau \geq \frac{1}{b} = \frac{\tau_*}{R_0 - 1}. \quad (17)$$

This equation provides the relationship between the scale of activation time and parameter  $R_0$ . One can see that this scale decreases as  $R_0$  increases.

From (14), (17) it follows that an “epidemic threshold” for the sensor network is simply  $\alpha\tau_*N > 1$  or in terms of the ‘basic reproductive number’ (11)

$$R_0 = \alpha\tau_*N = pN \frac{\pi r_*^2}{S} > 1. \quad (18)$$

Observe that sensor sampling time  $\tau_*$  has disappeared from the expression for  $R_0$ . This means that it is possible to create an information epidemic (i.e. detect a chemical pollutant) for any value of  $\tau_*$ , provided this time is long enough for a sensor to detect the chemical tracer. But according to (17), the responsiveness of the whole network to the external challenges (i.e. the time constant of detection) is, indeed, strongly dependent on the sensor sampling time  $\tau = \tau_*/(R_0 - 1)$ .

The expressions (16), (17) and (18) are the main analytical results of the paper. For a given level of external challenges (i.e.  $C_0$ ) and meteorological conditions (i.e.  $\gamma, \omega$ ), these expressions provide a simple yet rigorous way to estimate how a change in the network and sensor parameters (i.e.  $N, C_*, \tau_*$ ) will affect the network performance (i.e.  $N_+, \tau$ ). We can also see that for a given external challenge the network of chemical sensors will respond in the most effective way when its parameters are selected in the combination which meets the criterion for ‘information epidemic’ (18).

The final analytical expressions enable us to maximize the network information gain and optimize other parameters. For example, from (16), we can readily infer the important scaling properties of the network performance:

$$\frac{N_-}{N} \sim \frac{1}{r_*^2}, \quad \frac{N_-}{N} \sim \frac{1}{N}, \quad \frac{N_-}{N} \sim \frac{1}{p}. \quad (19)$$

For instance, if we double the communication range of an individual sensor  $r_*$ , the fraction of inactive sensors in the network will drop four times. Likewise, if we need to reach a specified fraction of active sensors  $(1 - N_-/N)$  to be able to reliably detect a given level of pollutant concentration, these formulas describe all possible ways of changing the parameters of the model in order to achieve this goal.

## 5. Information Gain of Collaboration

We have explained earlier that the concept of DSC is important for a network with limited energy/material resources. But the question remains will a network with DSC be inferior (in terms of detection performance) in comparison with a benchmark network where all sensors operate independently of each other and only report their (positive) detections of chemical pollution to the central processor for decision making? Clearly, such a benchmark network would be very expensive to run (all sensors would have to be active all the time), but could provide excellent detection performance.

In this section we show that, under a certain condition, the network with DSC can provide superior detection performance compared to the benchmark network. Let us assume that we have  $\delta N$  sensors continuously operating ( $0 \leq \delta \leq 1$ ). For a benchmark network, on average, we have  $p\delta N$  sensors detecting pollutant. For the network with DSC the same quantity can be estimated as  $p(1 - \theta)N$  (since as we have seen the saturation

level of  $N_+$  does not depend on initial conditions). From here we can then deduce that the network with DSC will provide more information (for detection of chemical pollution) than the benchmark network if the following condition is satisfied:

$$\theta = \frac{1}{\alpha\tau_*N} \leq (1 - \delta), \quad (20)$$

which is eventually reduced to the condition of “epidemic threshold”(18) for the small value of  $\delta$ .

The value of the parameter  $\delta$  can be also estimated based on the following arguments. Let us assume that our aim is to detect a level concentration  $C_0$  associated with a hazardous release within the time  $T$  (the constraint on time is driven by the requirement to mitigate the toxic effect of the release). Then we can write a simple condition for the information ‘epidemic’ in the WSN to occur during time  $T$ :

$$\delta pNT/\tau_* \geq 1, \quad (21)$$

where  $p$  is given by (5), i.e.  $p = 1 - F(C_*|C_0)$ . Evidently, for information epidemic to be observable, the number of continuously active sensors should be less than the number of sensors activated due to the hazardous release. Thus from (20) we can write the following ‘consistency’ condition for the minimum value of  $\delta$

$$\delta_{\min} \approx \frac{\tau_*}{pNT} \leq (1 - \frac{1}{\alpha\tau_*N}), \quad (22)$$

or by re-writing it in terms of  $R_0$ , see (16),

$$\delta_{\min} \approx \frac{\tau_*}{pNT} \leq (1 - \frac{1}{R_0}). \quad (23)$$

It can be seen, that with other conditions being equal the fraction of ‘stand-by’ sensors  $\delta_{\min}$  can be made however small (since  $R_0 \geq 1$ ). It implies that only a small fraction of WSN will be active most of the time and is a clear demonstration of the energy consumption gain associated with the ‘epidemic’ protocol.

Another important criteria for epidemic protocol can be derived by comparison of amplitude of “detectable events” for the *same number of sensors* in the network with DSC with the system of  $N$  independent sensors. For the network with DSC it is  $(1 - \theta)N$  (since we use  $N_+$  to retrieve information about the environment) and for the system of

the *same independent sensors* it is still  $pN$  (since  $N_+$  is simply equal to  $N$ ). Then instead of (20) we can write

$$\theta < (1 - p) \quad (24)$$

Under this condition more detectable events will occur in the presence of chemical pollution by the described network with DSC (activation messages) than in a network of stand alone sensors (signals of positive detection). This leads to the interesting threshold condition on the number of sensors in the network

$$N > \frac{S}{\pi r_*^2} \frac{1}{p(1-p)}. \quad (25)$$

The last term in RHS  $(p(1-p))^{-1}$  has an obvious minimum 4 corresponding to  $p = 1/2$ , so finally we arrive at the simple universal condition

$$N > N_* = \frac{4}{\pi} \frac{S}{r_*^2}. \quad (26)$$

This condition reads that if the number of sensors in the system is greater than  $N_*$  then networking with DSC *can* provide an information gain over the benchmark network. Under this condition, the network with DSC is not only desirable from the aspect of energy conservation, but also provides better detection performance through the information gain.

The condition  $p = 1/2$  minimizing RHS of (25) can be considered as a criterion for an “optimal” sensor for a given network with DSC and for a given concentration of pollutant to be detected. Namely, from the equation  $F(C_*|C_0) = 1/2$  and using (2) we can write

$$C_* = C_0 \left( \frac{\gamma - 2}{2} \right) \left[ \left( \frac{1}{2\omega} \right)^{1/(1-\gamma)} - 1 \right]. \quad (27)$$

Given environmental parameters  $(\gamma, \omega)$  and given the level of concentration to be detected ( $C_0$ ), formula (27) also specifies a simple condition on detection threshold for an individual sensor to maximize an information gain by being networked.

## 6. Numerical Simulations

In support of analytical derivations presented above, a network of chemical sensors operating according to the adopted protocol for dynamic collaboration was implemented

in MATLAB. A comprehensive report with numerical simulations result will be published elsewhere; here we present only some illustrative examples.

For consistency, a  $1000\text{m} \times 1000\text{m}$  surveillance region populated by  $N = 400$  sensors with a uniformly random placement was assumed in all tests. In each run, chemical pollution with concentration  $C_0 = 150$  is applied, and the simulation starts when a single randomly selected sensor (which has detected the presence of chemical contamination in its vicinity) starts broadcasting. Due to this random initiation and the fact that the probability of detection of individual sensors is less than unity ( $p < 1$ ), each run of the computer program results in a slightly different outcome. Figs.2,3 show the average evolution of the ratio  $N_+/N$  in the network over time. The curves were obtained by using the following parameters:  $\omega = 0.98$ ,  $\gamma = 26/3$ . Fig.2 demonstrates the changes in dynamics of the WSN for different values of communication range  $r_*$  and Fig.3 depicts the similar plots for changes of the detection threshold of individual sensor  $C_*$ . For all plots in Fig.2, Fig.3 the initial number of active sensors  $N_+(t = 0) = 10$ .

Overall we found that the simulation output is much more sensitive to the changes of communication range, than to the threshold of an individual sensor (see range of parameters depicted in Figs.2,3). In all cases we observed the transition of  $N_+$  from the initial steady state (where  $N_+$  is very small indicating the absence of the pollutant) to the new steady state (high value of  $N_+$ ), so information “epidemic” in the network of chemical sensors does occur. By direct substitution into (18) it was also validated that in all cases presented in Figs.2,3 the condition for an information “epidemic” was satisfied. In general, the saturation value of  $N_+$  derived from these plots were in an agreement with theoretical prediction (16), but the estimated standard deviation of  $N_+$  (not shown in Fig.2) could be very high (up to 30%) for some combination of parameters. The relative standard deviation (normalized by mean value  $N_+$ ) usually gradually decreased over time and quite rapidly decays with the increase of communication range  $r_*$ . The occasional high variability of the output of the sensor network is undesirable and motivates further analysis. We also used the data from the plots in Figs. 2, 3 to calibrate our model. The calibration was performed by extracting the steady-state (or saturation) values of  $N_+$  from the plots and by adjusting the “free” constant  $G$  in the analytical expressions (16) to achieve the best match between the analytical predictions and simulations. The

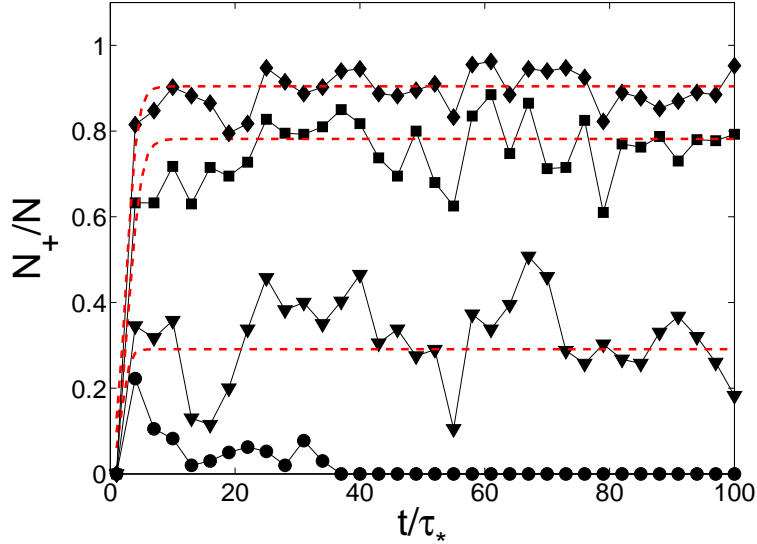


Figure 2: Results of numerical simulations: The fraction of active sensors in the network over time for the different communication range  $r_*$ :  $\diamond - r_* = 40m$ ,  $\square - r_* = 30m$ ,  $\nabla - r_* = 27m$ ,  $\circ - r_* = 20m$ ;  $C_*/C_0 = 1.03$ ,  $N_+(t=0) = 10$ . The dashed red line corresponds to the analytical predictions (15). It is clearly seen that in the case  $r_* = 20m$  the information epidemic in WSN dies off.

value  $G \approx 0.7$  seems to provide an optimal agreement with the presented simulations.

In order to validate our simple model for parameter  $\alpha$  we performed the following study. For each simulation we derived value of  $\alpha_s$  from (16) and then compared it with the value  $\alpha_t$  calculated from the theoretical expression (9) using the calibration value  $G \approx 0.7$ . The results of this study are presented in Fig.4. The red dashed line corresponds to the perfect agreement between the theory and simulations. Considering the high variability of  $N_+$  and a rather simple model for  $\alpha$ , the agreement between the theory and simulations is acceptable.

To validate further the alignment between the computer simulations and the proposed mathematical model, we numerically estimated some scaling properties of the network system (i.e. (9), (19)). Firstly we derived the scaling properties from computer simulations and then compared them to the theoretical predictions. In general we found that all trends of the derived scaling do agree with theoretical expressions in (19), but the quantitative agreement may significantly vary from case to case. As an illustration, in

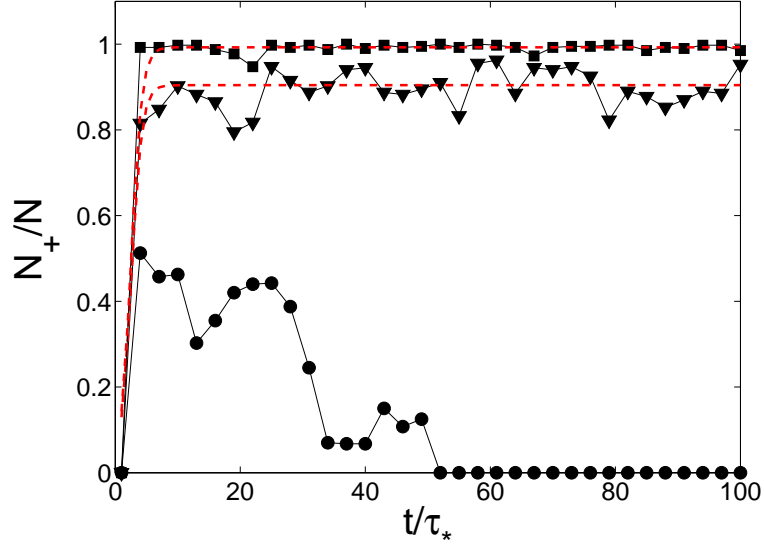


Figure 3: Results of numerical simulations: The fraction of active sensors in the network over time for the different threshold of individual sensor  $C_*$ :  $\square - C_*/C_0 = 1.05$ ,  $\nabla - C_*/C_0 = 1.02$ ,  $\circ - C_*/C_0 = 1.00$ ;  $r_* = 40m$ ,  $N_+(t = 0) = 10$ . The dashed red line corresponds to the analytical predictions (15). It is clearly seen that in the case  $C_*/C_0 = 1.00$  the information epidemic in WSN dies off.

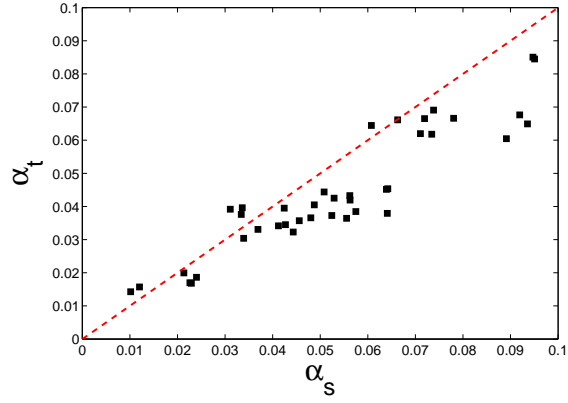


Figure 4: Simulation and theoretical predictions of parameter  $\alpha$  :  $\alpha_t$  is the theoretical value (9),  $\alpha_s$  is the results of simulations. The red dashed line corresponds to the perfect agreement.

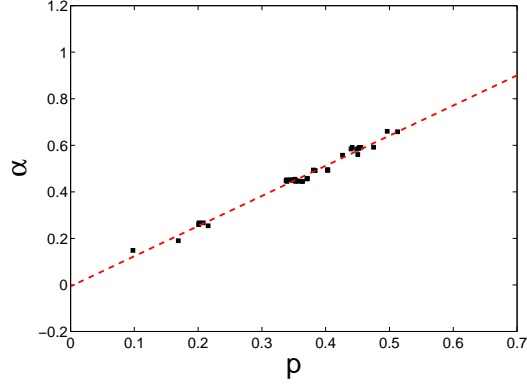


Figure 5: Parameter  $\alpha$  as a function of  $p$  extracted from numerical simulations in log-log scale. The dashed line corresponds to the power-law fit  $\alpha \propto p^q, q = 1.27$ , theoretical prediction corresponds to  $q = 1$ , see (8).

Fig.5 we present the plot of dependency of  $\alpha$  against  $p$  in log-log scale. The extracted exponent corresponds to  $\alpha \propto p^q$ , where  $q = 1.27$  while the theoretical value according to (8) is  $q = 1$ . This indicates that while our analytical model is very simple and fast to compute, for higher accuracy it may need further refinements as discussed below.

The results of numerical simulations presented above serve to verify that the ‘information epidemic’ does occur in the wireless network of chemical sensors. This also implies that the proposed theoretical framework may lead to a gain in the energy consumption, that may result in the significant advantages in operational deployment of such systems. More detailed analysis of the optimal values of parameters satisfying threshold conditions (18), (23), (26) and lead to the optimal performance of WSN will be reported in separate publications.

## 7. Refinements of the model

The disagreement described above is due to the implicit assumption of “homogeneous mixing” which we made in equations (6)-(7). The homogeneous mixing manifests itself in the bilinear form of the interaction terms on the RHS of (6)-(7). This bi-linearity means that the number of new “infected” sensors is proportional to the product of the number which is currently “infected” and the number which is currently “susceptible”. Effectively



it means that all passive sensors are equally likely to be activated. This assumption holds only if the majority of activated (“infected”) sensors are far away from each other (i.e. at the distances  $\gg r_*$ ). At some stage of the sensor “epidemic” this assumption can be violated, because the secondary “infected” sensors will be at the shorter distances from the “infectious” parents (see Fig.6). The broadcasted messages in overlapping areas become duplicated and the rate of new “infections” will be no longer proportional to the number of their parents. The fraction of “infected” sensors in the overlapping areas will depend on the new equilibrium state of the sensor system (i.e.  $N_+/N$  as  $t \rightarrow \infty$ ) and may not be small for some scenarios. To overcome this restriction we again invoke an approach successfully implemented in epidemiology (see [15]). Instead of (6)-(7) we now write

$$\frac{dN_+}{dt} = \alpha N_+^\nu N_- - \frac{N_+}{\tau_*}, \quad \frac{dN_-}{dt} = -\alpha N_+^\nu N_- + \frac{N_+}{\tau_*}, \quad (28)$$

where a new parameter  $0 \leq \nu \leq 1$  depends on the packing density of “infected” sensors (or on the ratio  $N_+/N$ ). For a “sparse” network configuration we have  $\nu \approx 1$  (no overlapping areas) and for an extremely “dense” network  $\nu \approx 0$  (all sensors are located around the same point), see Fig. 6. In general  $\nu$  can be used as a fitting parameter of the model [9] or estimated based on the mathematical theory of packing. For a specific network configuration a value  $\nu = 1/2$  was derived in [7] based on some simplified assumptions. By employing new parameter  $\nu$  we can significantly improve agreement between analytical model and simulation at the initial stage of information epidemic, since here we can assume  $N_- \approx N = \text{const}$ , so  $\frac{dN_+}{dt} \propto N_+^\nu$ . An example of improved fitting is presented in Fig.7.

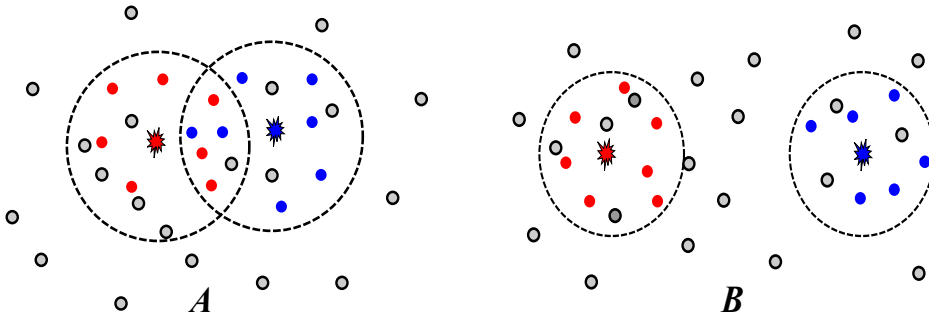


Figure 6: Examples of dense (A) and sparse (B) wireless sensor networks.

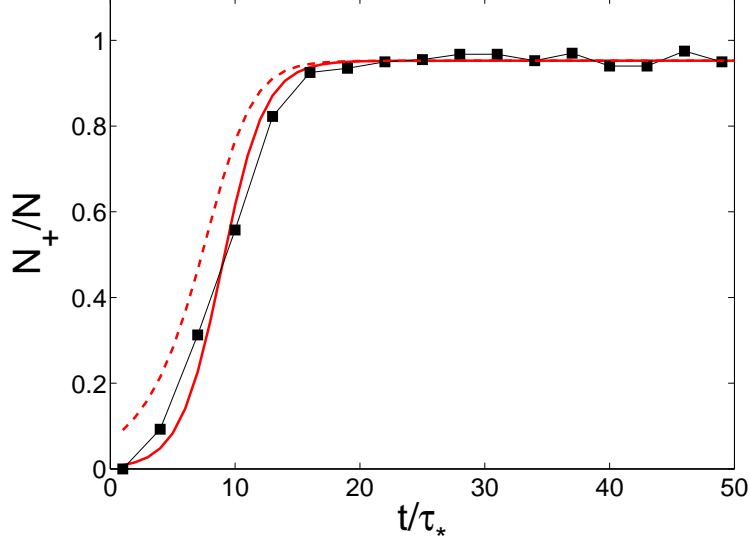


Figure 7: Effect of parameter  $\nu$  in the model (28) on the simulation data fit: dashed line  $\nu = 1$ , solid line  $\nu = 0.7$ .

Similarly to the epidemiological models (see [12]), incorporation of the spatial inhomogeneity can be achieved by adding the appropriate diffusion terms on the LHS of (6) and (7):

$$\frac{\partial N_+}{\partial t} - D\Delta N_+ = \alpha N_+ N_- - \frac{N_+}{\tau_*}, \quad (29)$$

$$\frac{\partial N_-}{\partial t} - D\Delta N_- = -\alpha N_+ N_- + \frac{N_-}{\tau_*}, \quad (30)$$

where  $D$  is diffusivity in the sensor system which can be estimated as  $D \approx r_*^2/\tau_*$ . At the same time the inhomogeneity of pollutant distribution can be easily incorporated in  $\alpha(\mathbf{r})$  with non-uniform  $C_0(\mathbf{r})$  (see (2), (5), (8)).

An important property of the system (29), (30) is the existence of analytical solutions in the form of traveling waves, propagating with the velocity  $v_0 \sim \sqrt{\alpha D}$  [12]. In our case these waves correspond to the switching fronts between active and passive sensors. If pollutant is advected by the wind flow with a characteristic velocity  $v_*$ , then a simple synchronisation condition  $v_0 \geq v_*$  or  $\alpha \geq v_*^2 \tau_*/r_*^2$  provides an important criteria for network optimisation.

Another interesting extension of the proposed model is the introduction of the concept of a *faulty* sensor, a sensor which is no longer available for sensing and networking. This state of a sensor would correspond to the *removed* population segment in the epidemiological framework and can be attributed to any kind of faults (flat battery, software malfunction, hardware defects etc). As in the celebrated SIR epidemiological model [12], a new state results in the third equation for  $N_0$  in the system (6)-(7) with a new temporal parameter - an average operational time (the lifespan) of a sensor. The total number of sensors will be still conserved:  $N = N_+ + N_- + N_0 = \text{const}$ . This model provides a more realistic representation of an operational sensor systems and allows us to estimate such important parameters as the operational lifetime of the network and the reliability of the network.

## 8. Conclusions

We developed a “bio-inspired” model of a network of chemical sensors with dynamic collaboration for the purpose of energy conservation and information gain. The proposed model leverages on the existing theoretical discoveries from epidemiology resulting in a simple analytical model for the analysis of network dynamics. The analytical model enabled us to formulate analytically the conditions for the network performance. Thus we found an optimal configuration which, within the underlying assumptions, yields a balance between the number of sensors, detected concentration, the sampling time and the communication range. The findings are partly supported by numerical simulations. Further work is required to address the model refinements and generalisations.

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## References

- [1] C. S. Raghavendra, K. M. Sivalingam, Taieb Znati. (2005) Wireless Sensor Networks. Springer, USA, 2005.

- [2] P. E. Bieringer, A. Wyszogrodzki, J. Weil and G. Bieberbach. An Evaluation of Propylene Sampler Grid Designs for the FFT07 Field Program(2006), Tech. Report, National Center for Atmospheric Research, Boulder, USA.
- [3] E. Ertin, J. W. Fisher, L. C. Potter.(2003) Maximum Mutual Information Principle for Dynamic Sensor Query Problems. *Lecture Notes in Computer Science: Information Processing in Sensor Networks*, 2003 **2634**, pp. 91-104.
- [4] F. Zhao, J. Shin, J. Reich. (2002) Information-Driven Dynamic Sensor Collaboration for Tracking Applications. *IEEE Signal Processing Magazine*, 2002, **19**, 2, pp. 61–72.
- [5] J. Mathieu, G. Hwang, J. Dunyak (2006). The State of the Art and the State of the Practice: Transferring Insights from Complex Biological Systems to the Exploitation of Netted Sensors in Command and Control Enterprises, *2006 MITRE Technical Papers*, July, 2006, MITRE Corporation, USA.
- [6] A. Khelil, C. Becker, J. Tian, K. Rothermel.(2002) An Epidemic Model for Information Diffusion in MANETs.In *MSWiM 2002: Proceedings of the 5th ACM international workshop on Modeling analysis and simulation of wireless and mobile systems*, Atlanta, Georgia, USA, 2002, pp. 54–60.
- [7] P. De, Y. Liu, S. K. Das. (2007) An Epidemic Theoretic Framework for Evaluating Broadcast Protocols in Wireless Sensor Networks. In *MASS 2007: Proceedings of IEEE International Conference on Mobile Adhoc and Sensor Systems*, Pisa, Italy, 2007, pp 1–9.
- [8] A. Gunatilaka, B.Ristic, A.Skvortsov, M. Morelande. (2008) Parameter Estimation of a Continuous Chemical Plume Source. in *Fusion 2008: 11th International Conference on Information Fusion*, Cologne, Germany, 2008, pp. 1-8.
- [9] B.Ristic, A.Skvortsov, M.Morelande. (2009) Predicting the Progress and the Peak of an Epidemics. in *ICASSP 2009: Proceedings of 2009 IEEE International Conference on Acoustics, Speech and Signal Processing*, Taiwan, April, 2009.
- [10] A.Dekker, A.Skvortsov. (2009) Topological Issues in Sensor Networks. in *MODSIM 2009: 2009 MSSANZ International Congress on Modelling and Simulation*, Cairns, Australia, 2009.
- [11] S. Eubank, V. S. Anil Kumar, M. Marathe. (2008) Epidemiology and Wireless Communication: Tight Analogy or Loose Metaphor? *Lecture Notes in Computer Science: Bio-Inspired Computing and Communication*, 2008, **5151**, pp. 91-104.
- [12] J. D. Murray.(2002) Mathematical Biology, Springer, USA, v 1,2. 2002
- [13] V. Bisignanesi, M.S. Borgas (2007). Models for integrated pest management with chemicals in atmospheric surface layers. *Ecological modelling*, 2007, **201**, 1, pp. 2–10.
- [14] P. D. Stroud, S. J. Sydoriak, J. M. Riese, J. P. Smith, S. M. Mniszewski, and P. R. Romero. (2006) Semi-empirical Power-law Scaling of New Infection Rate to Model Epidemic dynamics with Inhomogeneous Mixing, *Mathematical Biosciences*, **203**, pp. 301–318.
- [15] A.T. Skvortsov, R.B.Connell, P.D. Dawson, R.M. Gailis. (2007) Epidemic Spread Modeling: Alignment of Agent-Based Simulation with a Simple Mathematical Model. in *BIOCOMP 2007: Proceedings of International Conference on Bioinformatics and Computational Biology*, Las Vegas Nevada, USA, CSREA Press, 2007, **2**, pp 487–490.

- [16] A. Gunatilaka, A. Skvortsov, and R. Gailis. (2008) Progress in DSTO CBR simulation environment development, Land Warfare Conference (LWC2008), Brisbane, 2008, pp. 62–68.
- [17] A.Skvortsov, E.Yee. Scaling laws of peripheral mixing of passive scalar in a wall-shear layer (2008). *Phys. Rev. E* **83**, 036303–11.
- [18] M. Jamriska, T. C. DuBois, A. Skvortsov Statistical characterisation of bio-aerosol background in an urban environment (2011), <http://www.arxiv.org/pdf/1110.4184>